# *A new and easy way of demonstrating some* propositions *in* Euclid *by the learned Mr.* ------ Ash. *a Member of the* Philosophical Society *of* Dublin *for promoting natural knowledge.*

THe Preeminence of *Mathematical* knowledge, and the certainty of its way of reasoning are manifest from the few or no controversies between the *Professors* thereof (especially in pure unmixt *Mathematicks*;) and from the easy discovery of *paralogisms*. Some of the reasons of which certitude may be these: because *quantity*, the *object* about which it is conversant, is a sensible obvious thing, and consequently the *Idæas* we form thereof are clear and distinct, and dayly represented to us in most familiar instances; because it makes use of termes which are proper, adæquate, and unchangeable; its *axioms* and *postulata* also are very few and rational. It assigns such causes and generations of *Magnitudes* as are easily apprehended and readily admitted; it rejects all trifling in words and *Rhetorical schemes*, all conjectures, authorities, prejudices, and passion; Lastly so exquisite an order and method in *demonstrating* is observed, that no *proposition* is pretended to be proved, which does not plainly follow from what was before demonstrated, as is manifest in *Euclid's Elements*. Now as a farther instance of the evidence of *Mathematical Theoremes*, I believe it were not difficult to demonstrate any one of *Euclid's* independently from the rest, without precedent *Lemma's* or propositions; as an essay of which I will here subjoyn some of the most useful, and upon which the solution of most *problems*, especially *Algebraical* ones, do depend, and those also in the most various and different parts of *Geometry*, *viz*. concerning the properties of angles,, circles, triangles, squares, proportionalls, and solids. The *Propositions* which I will endeavour to *demonstrate* thus indepen-

dently shall be these; the 32*d*, and 47*th*, of the 1*st* book, most of the 2*d*, and 5*th* books, the 1*st*, and 16*th* of the 6*th*, with their *Corollaries*. In order to demonstrate the 32*d*, I suppose it known what is meant by an angle, triangle, circle, external angle, parallels, and that the measure of an angle is the arch of a circle intercepted between its sides, that a right angle is measured by a quadrant, and 2 right angles by a semicircle. I say then (in *Fig*. 1.) that in the triangle A B C, the external angle B C E is equall to the 2 opposite internal ones A B C, B A C; for let a circle be drawn, C being the Center, and B C the *radius*, and let C D be drawn parallel to A B, those 2 lines being alwayes æquidistant will both have the same inclination to any 3*d* line falling upon them, that is (by the definition of angle) they will make equal angles with it, for if any part of CD (for instance) did incline more to BC then did AB, upon that very account they would not be parallel, it follows therefore that the angles ABC, BCD are equal also BA C= D C E, because A E falls upon 2 parallels, but the external angle BCE= BCD + DCE which were before proved to be equall to ABC, BAC (Q. E. D.) hence may be infer'd as a *corollary*, that the 3 angles of every triangle are equal to 2 right ones, for the angles ACB + BCE are measured by a semicircle and therefore equal to 2 right angles; Corollaries also from hence are the 20*th* 22*d* and 31*st* of the 3*d* book which contain the properties of circles, whose deduction from hence being most natural and obvious, I omitt.

In order to demonstrate the 47*th*, I suppose the meaning of the terms made use of to be known; and that 2 angles or superficies are equal when one being put on the other, it neither exceeds, nor is exceeded: this being allowed, I say the sides about the right angle are either equal or unequal, if equal (as in *Fig*. 2.) let all the squares be described, the whole figure exceeds the square of the *Hypothenuse* BC by the 2 triangles M, U, and exceeds also the squares of the other 2 sides AB. AC. by the 2 trian-

 gles

gles ABC, and S; which exceſſes are equal, for M is equal to ABC, the 2 ſides about the right angle, being 2 ſides of a ſquare, upon AB by ſuppoſition equal to AC, and the 3*d* ſide equal to BC, therefore the whole triangles are equal. after the ſame manner S and U are proved to be equal, therefore the ſquare of CB is equal to the ſquare of the 2 other ſides Q E D.

But if the ſides be unequal (as in *Fig.* 3*d)* let the ſquare be deſcribed, and the *parallelogram* L Q compleated, the whole Figure exceeds the ſquare upon C, by 3 triangles X,R,Z, and exceeds alſo the ſquare LA, AD, by the triangle ABC, and the *Parallelogram* P. Q. which exceſſes I ſay are equal, for Z is equal to ABC, the ſide C=BC, CD=AC, the angle D=A, and OCD=BCA, which is manifeſt by taking the common angle ACO out of the 2 right angles BCO, ACD, therefore by ſuperimpoſition the whole triangles are equal. In like manner X is proved equal to ABC, alſo R; and the parrallelogram PQ to be double of the triangle ABC; thus the exceſſes being proved equal, the remainders alſo will be equal, *viz.* the ſquare of BC to the ſquare of AB, AC (Q. E. D) manifeſt *corollaries,* from hence are the 35*th* and 36*th* of the 3*d* book, alſo the 12*th* and 13*th* of 2*d*. And here I ſhall obſerve that by this *Method* of proving the 47. 1. *Eucl.* 'tis manifeſt that that propoſition may be demonſtrated otherwiſe then *Euclide* has done it, and yet without the help of proportions, which *Peletarius* denyed as poſſible.

The firſt 10 propoſitions of the 2*d* book are evidently demonſtrated, only by ſubſtituteing *ſpecies* or letters inſtead of lines, and multiplying them according to the tenor of the propoſition; thus to inſtance in one or two; in *Fig.* 4 call the whole line A, and its parts B and C therefore A=B + C and conſequently AA=BB + CC + 2 BC which is the very ſence of the 4*th* of the 2*d* book. Thus alſo (in *Fig.* 5) let a line be cut into equal parts F, F, and let another line S be added thereto, tis manifeſt that

4 FF

4FF + 4SF + 2SS = 2FF + 2FF + 2SS + 4SF,
which is the 10 propofition of the fame book,

Almoft the whole doctrine of *proportionals*, viz. permutation, inverfion, converfion, compofition, divifion of *Ratios*, and proportion *ex æquo*, and confequently the moft ufeful propofitions of the *5th* book are clearly demonftrated by one definiton, and that is of fimilar or like parts, which are faid to be fuch as are after the fame manner or equally contained in their wholes; thus (in *Fig. 6)* the Antecedents A and C are either equal to their confequents or greater, or lefs, if equal, the thing is manifeft, if lefs, then (by the definition of *proportionals)* A and C are like parts of B and E, therefore what proportions the whole B and E have to one another, the fame will A and C have, which is permutation, likewife E : C :: B : A which is inverfion; fo alfo if from *proportionals* you take like parts, the remainders will be proportional, whence converfion and divifion are demonftrated; and if to proportionals you add like parts, the wholes will ftill be *proportional*, which is Compofition &c. If the *antecedents* be greater then the *Confequents*, the *Confequents* will be like parts of them, and the demonftration exactly the fame with the former.

The firft of the *6th* book is proved by confidering the generations of *Parallelograms*, which are produced by drawing or multiplying the *perpendicular* upon the *bafis*, that is, takeing it fo often as there can parts and divifions in the *bafe*: therefore (in *Fig. 7)* the fame proportion that RX fingle, hath to NX fingle, the fame hath RX multiplyed by XZ, that is, repeated a certain number of times, to NX multiplied by XZ, that is, repeated the fame number of times; which is as much as to fay RX : NX :: par : RZ : par : NZ; now that this proportion alfo is true in oblique angled *parallelograms*, is proved, becaufe they are equal to rectangled ones upon the fame *bafis* and between the fame *Parallels*, as does this independently appear *(in Fig. 7)* the triangles RQX and MPZ are equal, for RX=MZ, QX=PZ, RM=QP, therefore

adding

adding to both MQ, RQ=MP, if therefore from these equal triangles you take what is common *viz*. MLQ, the remainders will be equal RXLM=QLZP; to both which add XLZ, and the whole parallelograms will be equal, RZ=QZ (Q. E. D.) that triangles also having a common *basis*, are in the proportion of their altitudes does hence follow, because they are the halfs of *parallelograms* upon the same *basis*, this also is true, and the demonstration exactly the same in prisms, Pyramids, Cylinders, and cones, having the same basis.

To prove the 16*th* of the 6*th* I suppose (in *Fig*. 6.) the 4 lines A, B, C, E. to be proportional, that is, granting A and C to be the lesser terms, the same way that A is contained in B, so is C in E, and that D is the *denominator* of the *ratio*, 'twill follow then that B is made up of A, multiplied by D, and E of C multiplied by D, so that AD=B, and CD=E, draw therefore the extremes upon one another, that is A upon CD and the meanes, that is, C upon AD, the factors being the same, I say the products ACD and CAD are the same and consequently equal (Q. E. D.

I know not whether it be worth the while to add somewhat (tho altogether impertinent to this present subject) concerning *Mons: Comier's probleme* which he lately proposed with ostentation enough to all *Mathematicians* to be solved, as if it contained something new, whereas tis no more then the old business of *doubling* the *Cube* a little disguised, this has been shewn by several, but by none (I think) after the *algebraical* way, or so briefly as follows in *Fig*. 8.

$$A : 2X :: X : \frac{2Xq}{A} = P \qquad \text{per 8. 6.}$$

$$Aq + 2AX - \frac{2Xc}{A} = \frac{4Xqq}{Aq} \qquad \text{per 47. 1.}$$

$$Aqq + 2AcX - 2XcA = 4Xqq$$

$$Aqq + 2AcX = 4Xqq + 2XcA$$ resolving which *æquation*

$$A + 2X : 2A + 4X :: Xc : Ac :$$ into an *Analogy*.

that is, Xc (the *cube* upon X) is $\frac{1}{2}$ of Ac (the *cube* upon A)

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Fig: 1.

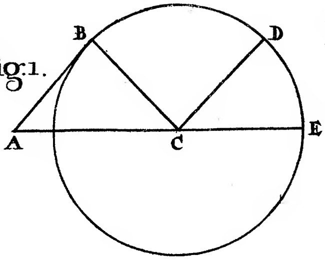




Fig: 4

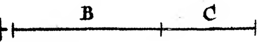




Fig: 5



Fig: 6.

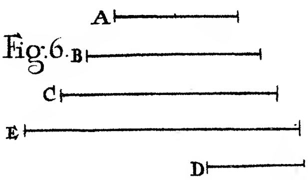




Fig: 2.

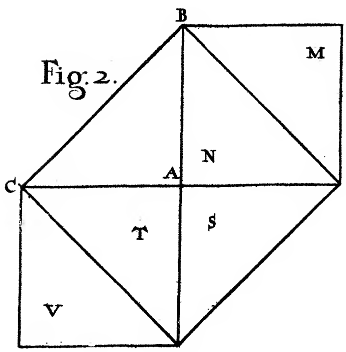




Fig: 7.

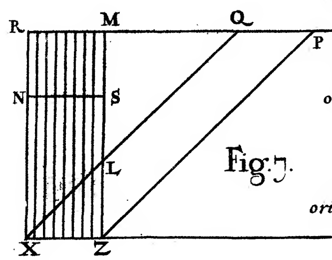




Fig: 9.

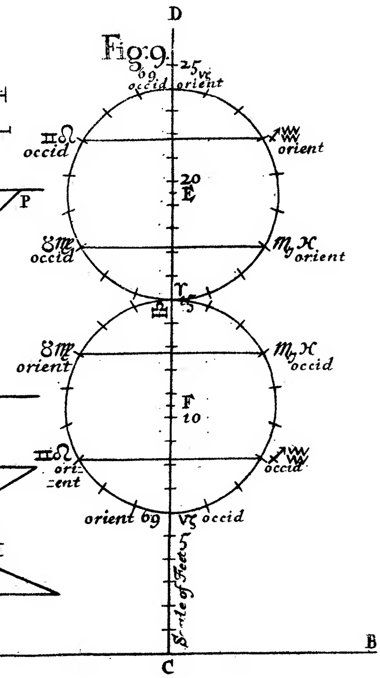




Fig: 3.

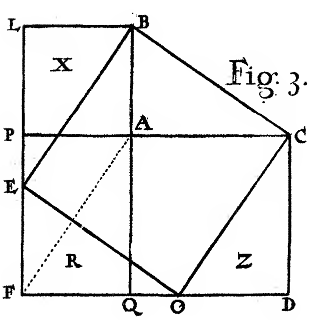




Fig: 8.

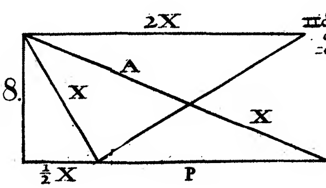




Fig. 10

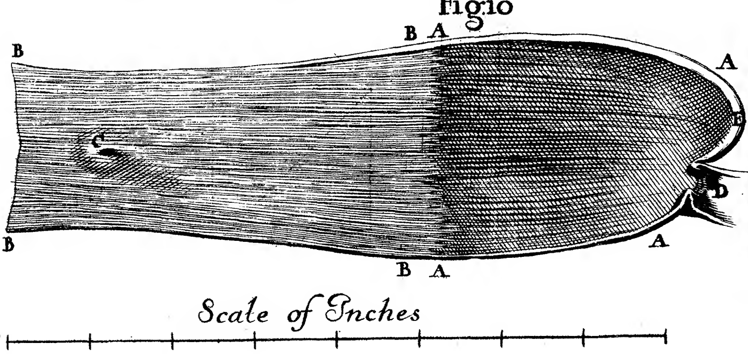
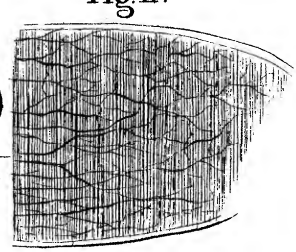




Fig. 11.



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